Supplementary material for

"Development of a novel computational model for the Balloon Analogue Risk Task: The Exponential-Weight Mean-Variance Model"

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Demographic and Clinical characteristics of the participants

	HC (N=135)		Her (N=47)		Amp (N=44)		Test Statistic (F)	Sig.	Post-hoc
	Mean	SD	Mean	SD	Mean	SD			
Demographic									
Age	26.99	6.14	31.60	5.97	25.05	5.41	15.26	6.41E-07	a, c
Gender (%male)	59.26		72.34		63.64		1.28	n.s.	
Education (years)	14.64	2.66	13.09	2.30	13.36	1.98	9.09	1.60E-04	a, b
IQ	110.08	12.98	105.96	10.84	109.00	12.58	1.90	n.s.	
# of relatives with alcohol/drug problems	0.44	0.83	0.87	1.20	0.71	0.92	3.96	0.021	a
Psychiatric									
History of conduct disorder (%)	2.22		25.53		18.18		13.59	2.70E-06	a, b
History of antisocial personality disorder (%)	0.00		23.4		18.18		18.11	5.15E-08	a, b
Years of heroin use	0.00	0.00	6.18	3.18	0.33	2.13	234.3	2.00E-16	a, c
Years of amphetamine use	0.00	0.00	0.07	0.25	4.27	2.87	181.3	2.00E-16	b, c
Years of alcohol use	9.52	5.84	13.39	5.86	8.37	4.65	10.38	4.98E-05	a, c
DSM-IV past dependence									
Alcohol (%)	0.00		0.00		0.00				
Sedatives (%)	0.00		0.00		0.00				
Cannabis (%)	0.00		0.00		0.00				
Stimulants (%)	0.00		0.00		100.00				
Opiates (%)	0.00		100.00		0.00				
Cocaine (%)	0.00		0.00		0.00				
Hallucinogens (%)	0.00		0.00		0.00				
Length of abstinence (years)	2.64	3.64	3.96	5.31	0.68	2.04	7.91	5.54E-04	b, c
Fagerstrom test of nicotine dependence	1.37	2.27	3.15	2.62	2.70	2.63	11.76	1.39E-05	a, b
Psychopathy Checklist (PCL:SV) Factor 1	1.47	1.58	4.87	2.81	3.41	2.75	48.02	2.00E-16	a, b, c
Psychopathy Checklist (PCL:SV) Factor 2	1.39	1.77	6.44	2.69	4.66	2.63	107.5	2.00E-16	a, b, c
Wender Utah Rating Scale (WURS) for ADHD	19.93	11.21	30.17	16.94	28.27	15.79	13.14	4.03E-06	a, b
Beck Depression Inventory (BDI-II)	4.83	4.73	7.76	6.37	6.77	5.74	6.15	0.003	a
State anxiety	31.49	7.31	35.83	10.11	32.25	7.15	5.25	0.006	a
Trait anxiety	36.75	9.03	39.96	10.54	38.30	9.20	2.14	n.s.	
Anxiety sensitivity	14.77	8.21	17.02	8.32	17.82	8.16	2.90	n.s.	

Table S1. Demographic, clinical characteristics of participants. n.s. indicates non-significant ($p > 0.05$). Test statistic and Sig. results are based on one-way ANOVA tests. Post-hoc results are based on the Bonferroni method.

HC: healthy control group, Her: heroin-dependent group, Amp: amphetamine-dependent group.

[a] HC vs. Her groups are significantly different.

[b] HC vs. Amp groups are significantly different.

[c] Her vs. Amp groups are significantly different.

Behavioral analysis

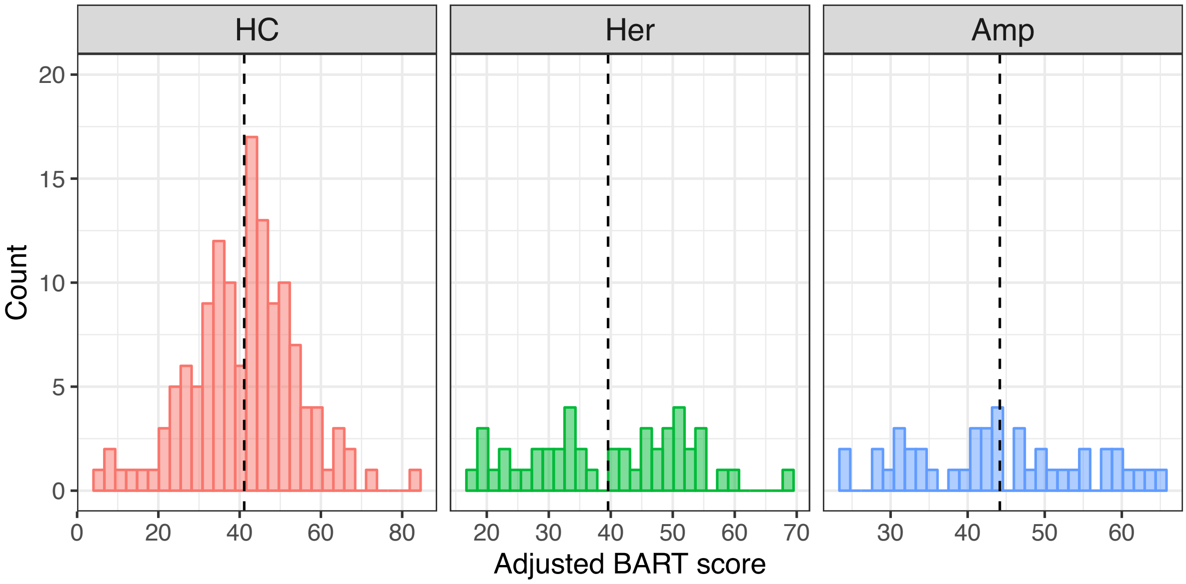

Figure S1. Distributions of the adjusted BART score (the average number of pumps for unexploded balloons) for each group. The black dotted vertical lines indicate the mean values. HC: healthy control group, Her: heroin-dependent group, Amp: amphetamine-dependent group.

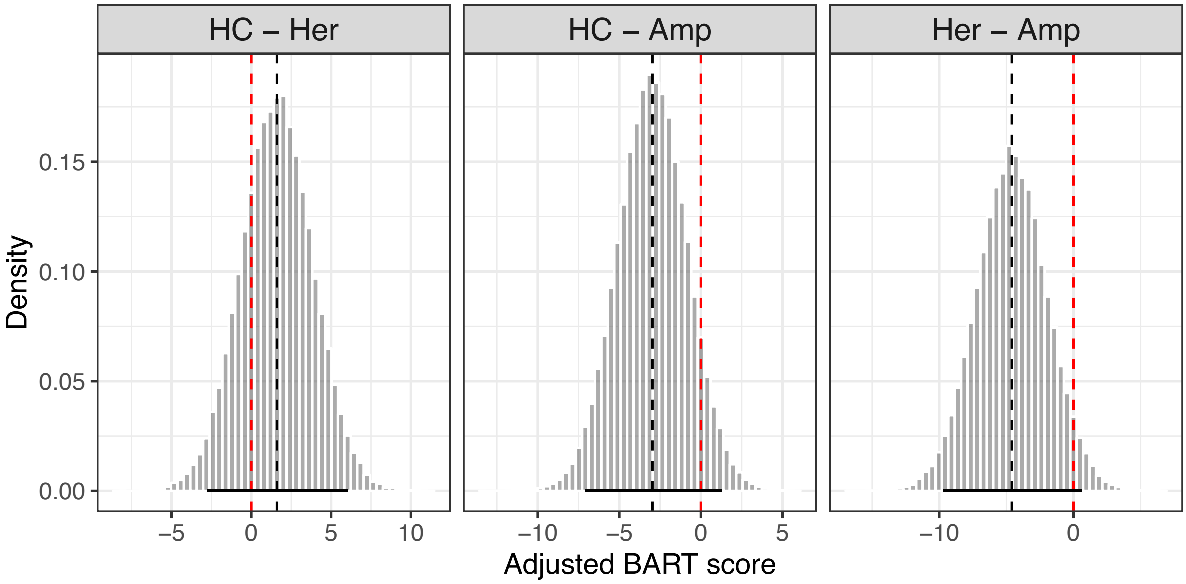

Figure S2. Group difference in the adjusted BART score (the average number of pumps for unexploded balloons). The black horizontal lines indicate 95% highest density intervals (HDIs). The red dotted vertical lines indicate $\boldsymbol{x} = \mathbf{0}$. The black dotted vertical lines indicate the mean values. HC: healthy control group, Her: heroin-dependent group, Amp: amphetamine-dependent group.

Computational modeling

Diagnostics for MCMC samples

We applied each model to data from each group separately. To check the quality of the MCMC samples, we used $\hat{R}$ values, trace plots, and Autocorrelation function (ACF) plots. All results for models and groups indicated that the MCMC chains were converged to stationary target distributions. Here, we show statistics and plots of the healthy control group for the EW model as an example (Table S2, Figure S3, and Figure S4).

Group parameter	Mean	Standard Deviation	2.5%	97.5%	Effective sample size	$\hat{R}$
ψ (prior belief of burst)	0.00749	6.79E-04	0.00618	0.00883	1023	1.01
ξ (updating exponent)	0.00456	7.67E-04	0.00318	0.00615	853	1.00
ρ (risk preference)	0.766	0.0350	0.700	0.836	1895	1.00
τ (Inverse temperature)	9.19	0.363	8.53	9.94	1560	1.00
λ (loss aversion)	4.20	0.597	3.08	5.39	1854	1.00

Table S2. Statistics of posterior distributions of group parameters for the healthy control group. The effective sample size for each parameter is the number of independent samples with the same estimation power as the total autocorrelated samples (Carpenter et al., 2017). The result shows that all parameters include an adequate effective sample size. $\hat{\boldsymbol{R}}$ values (Gelman & Rubin, 1992) for all parameters are close to 1.00, which indicates that the estimated parameter values converged to their target posterior distributions.

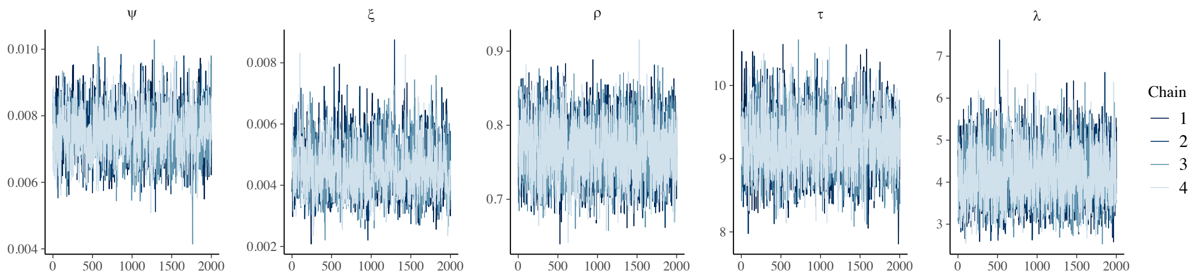

Figure S3. Trace plots of group parameters for the healthy control group. Consistent with the $\hat{R}$ values, the trace plots indicate that the MCMC samples are well mixed and converged. Note that the plots exclude burn-in samples.

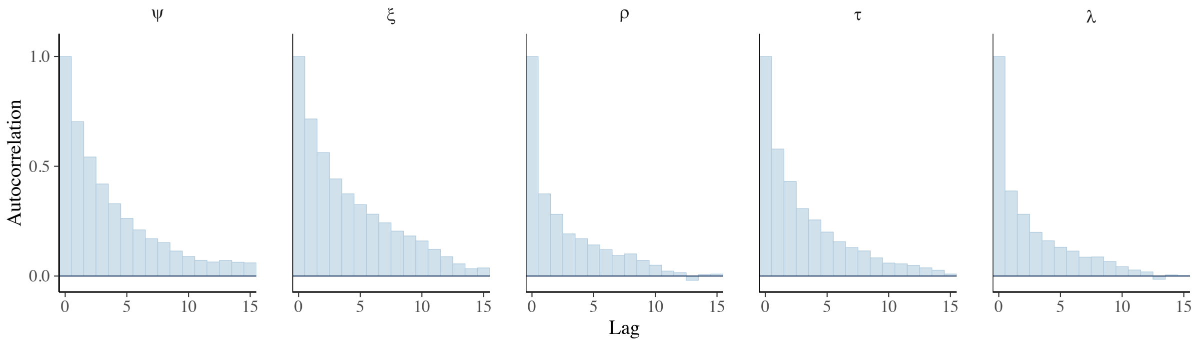

Figure S4. Autocorrelation function (ACF) plots of group parameters for the healthy control group. To reduce autocorrelation, thinning of MCMC chains can be applied, but it is not appropriate for precise estimates from MCMC samples (Link & Eaton, 2012). Accordingly, we did not use thinning for the sampling process.

Model comparison

Leave-one-out information criterion weight (LOOIC weight)

LOOIC weights are defined as Akaike weights (Wagenmakers & Farrell, 2004) calculated based on LOOIC values. To compute LOOIC weights, first, we need to compute differences in LOOIC between each model and the best-fitting model (a model with a minimal LOOIC value):

$$\Delta_i(LOOIC) = LOOIC_i - LOOIC_{\min}.$$

Based on the difference in LOOIC, we can obtain an estimate of the relative likelihood (L) for each model (M_i),

$$L(M_i|data) \propto \exp\left\{-\frac{1}{2}\Delta_i(LOOIC)\right\}.$$

Lastly, the LOOIC weights, $w_i(LOOIC)$, are obtained by normalizing the relative likelihoods,

$$w_i(LOOIC) = \frac{\exp\left\{-\frac{1}{2}\Delta_i(LOOIC)\right\}}{\sum_{n=1}^{N}\exp\left\{-\frac{1}{2}\Delta_n(LOOIC)\right\}}.$$

Parameter recovery

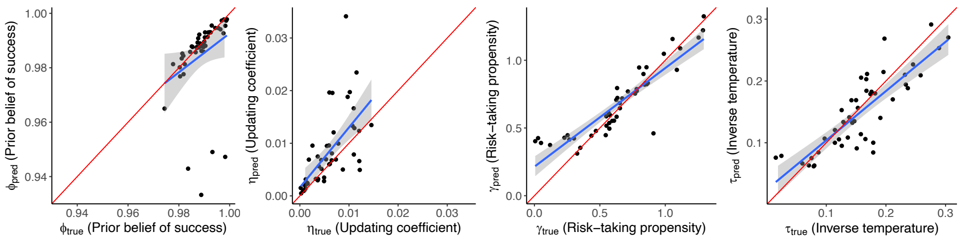

Figure S5. Parameter recovery results for the reparametrized version of the four-parameter model (Par4 model) from the heroin-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ϕ (prior belief of success): [0.298, 0.718, 0.275], η (updating coefficient): [0.643, 1.141, 0.002], γ (risk-taking propensity): [0.902, 0.728, 0.214], τ (inverse temperature): [0.840, 0.797, 0.024]. The average of the correlation coefficients is 0.671.

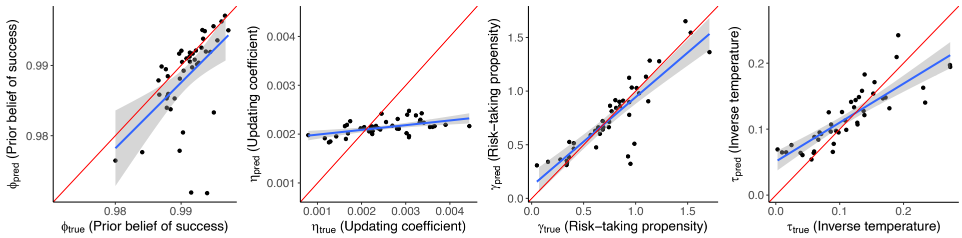

Figure S6. Parameter recovery results for the reparametrized version of the four-parameter model (Par4 model) from the amphetamine-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ϕ (prior belief of success): [0.526, 0.936, 0.060], η (updating coefficient): [0.514, 0.097, 0.002], γ (risk-taking propensity): [0.864, 0.829, 0.115], τ (inverse temperature): [0.846, 0.590, 0.051]. The average of the correlation coefficients is 0.687.

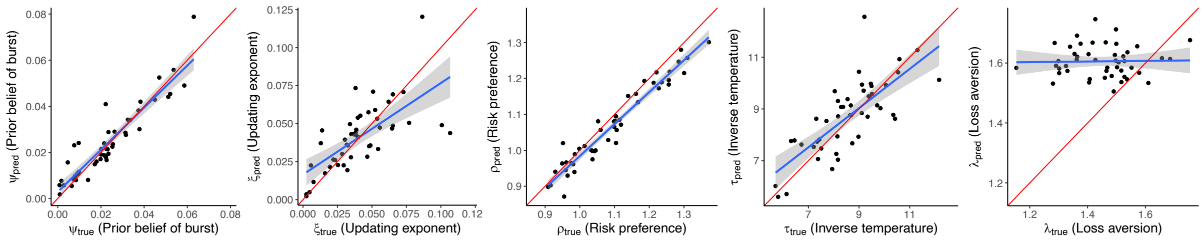

Figure S7. Parameter recovery results for the exponential-weight model (EW model) from the heroin-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ψ (prior belief of burst): [0.924, 0.914, 0.003], ξ (updating exponent): [0.681, 0.602, 0.017], ρ (risk preference): [0.968, 0.889, 0.095], τ (inverse temperature): [0.754, 0.758, 2.208], λ (loss aversion): [0.024, 0.010, 1.590]. The average of the correlation coefficients is 0.670.

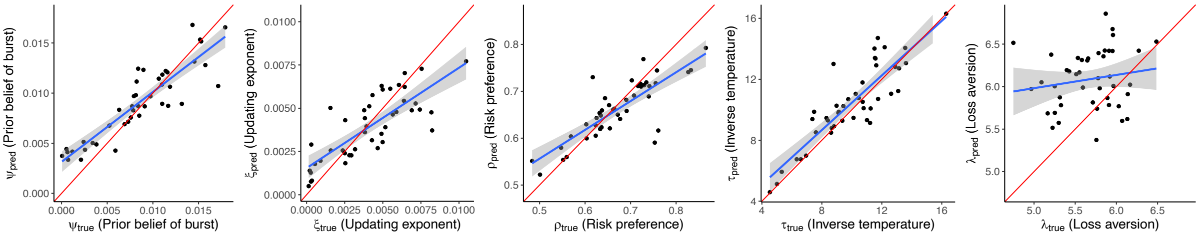

Figure S8. Parameter recovery results for the exponential-weight model (EW model) from the amphetamine-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ψ (prior belief of burst): [0.900, 0.697, 0.003], ξ (updating exponent): [0.794, 0.580, 0.002], ρ (risk preference): [0.815, 0.614, 0.249], τ (inverse temperature): [0.871, 0.896, 1.505], λ (loss aversion): [0.169, 0.156, 5.204]. The average of the correlation coefficients is 0.710.

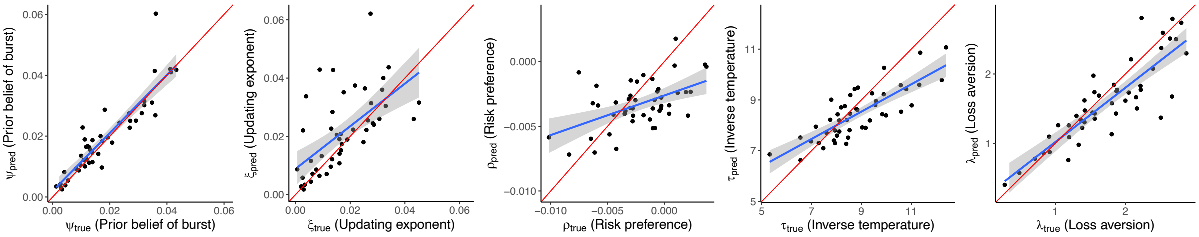

Figure S9. Parameter recovery results for the exponential-weight mean-variance model (EWMV model) from the heroin-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ψ (prior belief of burst): [0.907, 0.954, 0.002], ξ (updating exponent): [0.613, 0.731, 0.009], ρ (risk preference): [0.508, 0.306, -0.003], τ (inverse temperature): [0.801, 0.538, 3.698], λ (loss aversion): [0.860, 0.784, 0.235]. The average of the correlation coefficients is 0.738.

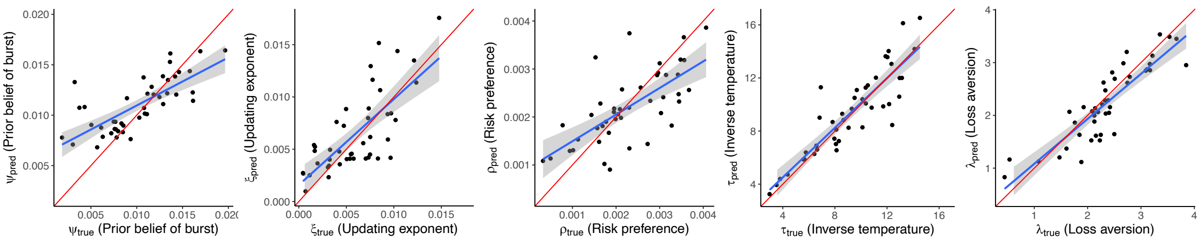

Figure S10. Parameter recovery results for the exponential-weight mean-variance model (EWMV model) from the amphetamine-dependent group. The red lines denote $y = x$. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals. The correlation and regression coefficients of each scatter plot is as follows [correlation, slope, intercept]. ψ (prior belief of burst): [0.745, 0.476, 0.006], ξ (updating exponent): [0.737, 0.821, 0.002], ρ (risk preference): [0.687, 0.555, 0.001], τ (inverse temperature): [0.885, 0.934, 0.774], λ (loss aversion): [0.877, 0.852, 0.230]. The average of the correlation coefficients is 0.786.

Correlation analysis

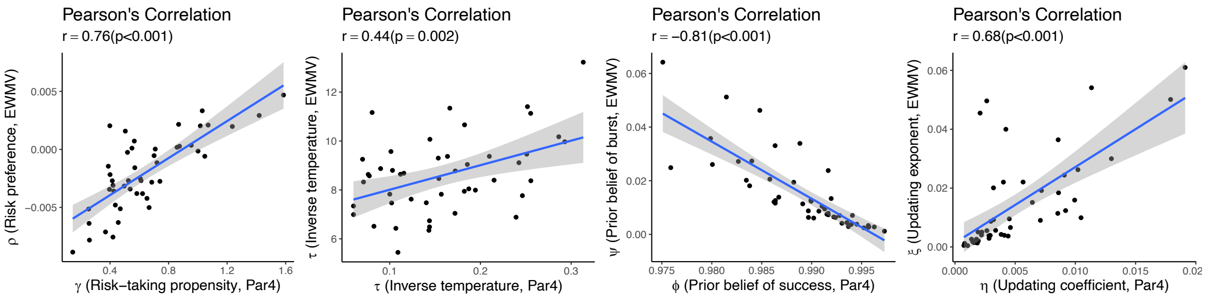

Figure S11. Correlations between the corresponding parameter pairs of the models from the heroin-dependent group. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals.

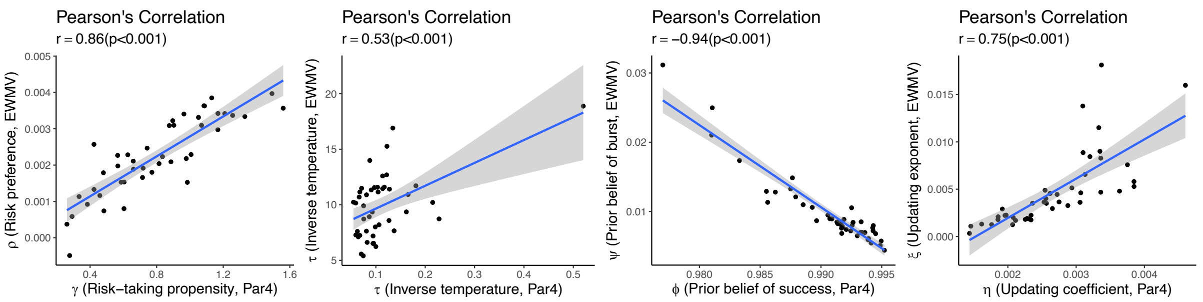

Figure S12. Correlations between the corresponding parameter pairs of the models from the amphetamine-dependent group. The blue lines indicate the regression lines of each graph. Shaded regions indicate 95% confidence intervals.

Group difference of model parameters

To compare model parameters for the three groups in a Bayesian fashion, we fitted each group with each model separately and calculated the posterior distributions of differences of group mean parameters (Ahn et al., 2014). The below figures show credible group differences (the 95% HDIs of the posterior distributions of group mean differences do not include zero) with the EWMV model (Figure S9) and the Par4 model (Figure S10).

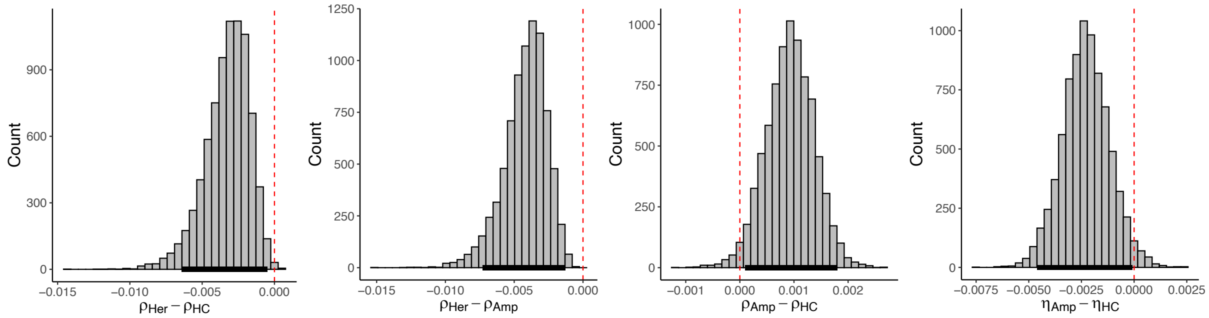

Figure S13. Posterior distributions of differences of group mean parameters with the exponential-weight mean-variance model (EWMV model). The figures show the posterior distributions of group mean differences for the credible group differences (the 95% HDIs of the posterior distributions of group mean differences do not include zero). The black horizontal lines indicate 95% highest density intervals (HDIs). The red dotted vertical lines indicate $x = 0$. HC: healthy control group, Her: heroin-dependent group, Amp: amphetamine-dependent group.

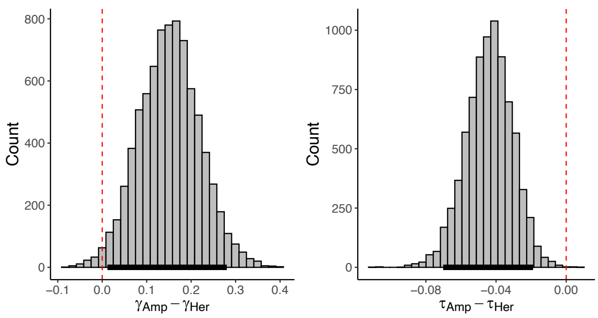

Figure S14. Posterior distributions of differences of group mean parameters with the reparametrized version of the four-parameter model (Par4 model). The figures show the posterior distributions of group mean differences for the credible group differences (the 95% HDIs of the posterior distributions of group mean differences do not include zero). The black horizontal lines indicate 95% highest density intervals (HDIs). The red dotted vertical lines indicate $x = 0$. HC: healthy control group, Her: heroin-dependent group, Amp: amphetamine-dependent group.

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